Why is getting the PAF right important?

James Collins

March 13, 2020

Suppose we introduce correlated risk factors but don't properly adjust for the changes this causes in the joint paf. How wrong will incidence be? For two risk factors, we calculate the incidence for an individual as

$$i = i_{GBD} \cdot (1 - PAF_{joint}) \cdot RR_1(e_1) \cdot RR_2(e_2)$$

Say our lack of adjustment causes us to underestimate the paf by 5%. So that $PAF_{joint} = 0.95 PAF_{true}$. Then

$$i = i_{GBD} \cdot (1 - 0.95 PAF_{true}) \cdot RR_1(e_1) \cdot RR_2(e_2)$$

We can see what impact this has at the population level by taking an expected value over the joint exposure distribution.

$$\begin{aligned}
E[i] &= E[i_{GBD} \cdot (1 - 0.95 PAF_{true}) \cdot RR_1(e_1) \cdot RR_2(e_2)] \\
&= i_{GBD} \cdot E[(1 - 0.95 PAF_{true}) \cdot RR_1(e_1) \cdot RR_2(e_2)] \\
&= i_{GBD} \cdot E[(1 - PAF_{true}) \cdot RR_1(e_1) \cdot RR_2(e_2) + 0.05 PAF_{true} \cdot RR_1(e_1) \cdot RR_2(e_2)] \\
&= i_{GBD} \cdot (E[(1 - PAF_{true}) \cdot RR_1(e_1) \cdot RR_2(e_2)] + E[0.05 PAF_{true} \cdot RR_1(e_1) \cdot RR_2(e_2)])
\end{aligned}$$

By definition $E[(1 - PAF_{true}) \cdot RR_1(e_1) \cdot RR_2(e_2)] = 1$ so this becomes

$$\begin{aligned}
E[i] &= i_{GBD} \cdot (1 + E[0.05 PAF_{true} \cdot RR_1(e_1) \cdot RR_2(e_2)]) \\
&= i_{GBD} \cdot (1 + 0.05 PAF_{true} \cdot E[RR_1(e_1) \cdot RR_2(e_2)])
\end{aligned}$$

Again, by definition, we can say $E[RR_1(e_1) \cdot RR_2(e_2)] = \frac{1}{1 - PAF_{true}}$ so we have

$$E[i] = i_{GBD} \cdot \left(1 + 0.05 \frac{PAF_{true}}{1 - PAF_{true}}\right)$$

How big is that? Well, it clearly depends on the PAF.

Absolute PAF	**Absolute PAF Error**	**Incidence Error (%)**
0.01	0.0005	0.05
0.10	0.0050	0.56
0.25	0.0125	1.67
0.50	0.0250	5.00
0.75	0.0375	15.00
0.90	0.045	45.00

Table 1: Incidence error due to 5% error in PAF